Domain wall fermions and the strange quark mass *

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The strange quark mass has been computed using a lattice action which possesses continuum–like chiral symmetry to good precision, namely the domain wall fermion action. This talk surveys this action and the recent calculation of m_s by the RIKEN/BNL/CU collaboration. This result is put into context by briefly summarizing other recent lattice studies.

1. INTRODUCTION

The strange quark mass is a fundamental parameter of the Standard Model, one whose value is poorly known. The two methods which have been used extensively to determine m_s are QCD sum rules and lattice QCD simulations. Before one can compare results from these different methods fairly, a basic understanding of each method's systematic uncertainties and approximations is necessary. This talk is focused on recent lattice calculations of m_s emphasizing such uncertainties. The calculation of m_s using domain wall fermions by the RIKEN/BNL/Columbia (RBC) lattice collaboration [1] is taken as a case study. The details of this work were presented at Lattice 99 [2], and there was another description of the relevant lattice techniques here by Becirevic [3].

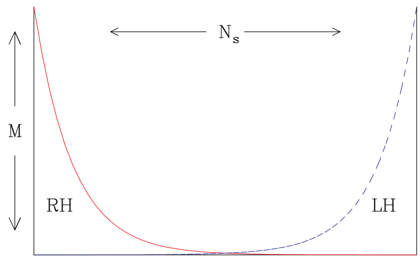

Figure 1. Schematic drawing of the light fermion wavefunctions in the extra dimension. For appropriate choice of the domain wall height M, a right–handed (RH) mode appears on the left boundary and a left–handed (LH) mode on the right boundary, N_s sites away. The mixing between the two modes within the extra dimension decays exponentially as N_s increases.

2. DOMAIN WALL FERMIONS

Symmetries are of fundamental importance in physics; in the case of QCD chiral symmetry is responsible for much of light hadron dynamics. However, chirally symmetric discretizations of the Dirac operator suffer from a "doubling" of the particle spectrum. For example replacing ∂_μ with a nearest–neighbor difference operator leads to the free quark propagator $S(p) = [i\gamma_\mu \sin(p_\mu a) + m]^{-1}$ which, in the massless limit, has poles at all corners of the Brillouin zone $p_\mu \in [0,\pi]$. Wilson added to the lattice action a second derivative–like operator which breaks chiral symmetry and gives the doubler states a mass inversely proportional to the lattice spacing. Another approach, by Kogut and Susskind (KS), is to stagger the spin degrees of freedom and interpret the doublers as different quark flavors. Although this method maintains a $U(1)$ remnant of the continuum chiral symmetry, flavor symmetry is badly broken. Both actions recover the symmetries of continuum quarks only in the $a \to 0$ limit.

An exciting and fruitful suggestion for decoupling the chiral and continuum limits started with Kaplan [4] who pointed out to the lattice

*Talk presented at QCD 99, Montpellier, France community that a chiral $2k$–dimensional mode is bound to a mass defect (or *domain wall*) in a $2k+1$–dimensional space. Specifically, in Shamir's boundary fermion variant [5,6], there is a massless right–handed (RH) mode stuck to the $4d$ boundary of a semi-infinite $5d$ space, given an appropriate choice for the $5d$ mass M. When the fifth dimension is made finite, a left–handed (LH) mode appears on the other boundary (see Fig. 1). The wavefunctions of the light modes decay exponentially within the fifth dimension, so there is a residual mixing between the RH and LH states which decreases as one increases N_s, the number of sites in the fifth dimension. This coupling of chiral modes describes a Dirac fermion with mass $m_{\rm res}$. In order to have control over the mass of the light Dirac fermion, the RH and LH boundaries are explicitly coupled with a weight $-am$. For $am \gg am_{\rm res}$ the mixing within the bulk of the fifth dimension is negligible.

The important feature of the domain wall (DW) fermion action is that the continuum limit $a \to 0$ is decoupled from the chiral limit $N_s \to \infty$ at the expense of an extra dimension of size N_s. Simulations with DW fermions have shown that for lattice spacings roughly 0.1 fm, $N_s = 10-20$ is sufficient for lattice QCD to exhibit continuum–like chiral properties, e.g. Ward identities [7] and suppression of wrong–chirality operator mixings [8].

3. RESULTS OF RBC SIMULATIONS

There are two ingredients which contribute to the determination of quark masses from the lattice: the calculation of hadron masses, decay constants, and/or matrix elements; and the calculation of the quark mass renormalization constant.

In Refs. [2,9] the pseudoscalar meson, vector meson, and nucleon masses are computed as functions of the bare quark mass, am. The am corresponding to the light quark mass is determined by extrapolating $(aM_{\rm PS}/aM_{\rm V})^2$ to $(M_\pi/M_\rho)^2$, and then the lattice spacing a can be defined by setting the ρ or N to its physical value. Finally the bare am corresponding to the strange quark mass is fixed by interpolating the pseudoscalar mass to M_K or the vector mass to M_{K^*} or M_ϕ.

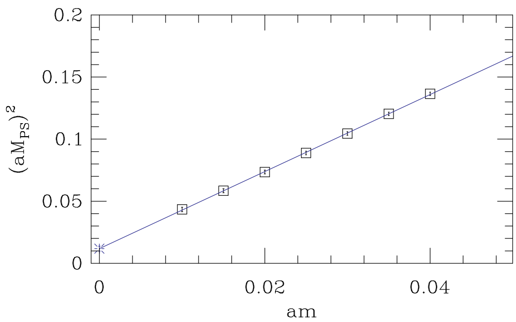

Figure 2. Pseudoscalar meson mass squared vs. am.

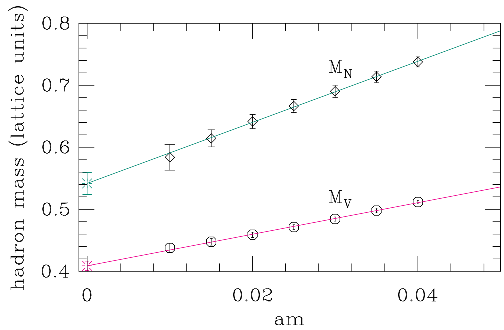

Figure 3. Vector meson and nucleon masses vs. am.

Figs. 2 and 3 show these hadron masses as functions of am. Due to a combination of finite spatial volume effects, finite N_s effects, and quenching errors, the chiral limit is at $am = -0.0038(6)$. Interpolating in the hadron masses as described above, one finds (with statistical errors only) [2]:

bare	$1/a = 1.91(0.04)$ GeV	
mass	lattice units	MeV
m_l	0.00166(0.00005)	3.17(0.11)
$m_s(K)$	0.042(0.003)	80(6)
$m_s(\phi)$	0.053(0.004)	101(7)

The scale above has been defined by setting the ρ mass to its experimental value; however $M_N/M_\rho = 1.37(5)$ compared to the experimental ratio 1.22, so there is at least a 10% uncertainty due to the ambiguity in whether the nucleon or ρ is used to set the scale. This is common among quenched lattice simulations with lattice spacings $O(0.1\text{fm})$.

In Ref. [2] the renormalization constant is computed nonperturbatively using the regularization independent momentum subtraction (RI/MOM) scheme as advocated by the Rome–Southampton group [10], applied for the first time to DW fermions [8]. The result at 2 GeV for $Z_m^{\overline{\rm MS}} \equiv 1/Z_S^{\overline{\rm MS}}$ is 1.63 ± 0.07 (stat.) ± 0.09 (sys.) which should be compared to 1.32, the one–loop perturbative Z_m [11,12] for these simulation parameters. One inherent assumption of this method is that perturbation theory is reliable at the scale of the inverse lattice spacing, roughly 2 GeV here. As Chetyrkin pointed out [13] the convergence of the perturbation series for the matching between RI/MOM and $\overline{\rm MS}$ schemes is slow at 2 GeV. The convergence would be much improved for $1/a \approx 2.5 - 3$ GeV.

The result from Ref. [2] at this lattice spacing (corresponding to $6/g_0^2 = 6.0$) is

$$m_s^{\overline{\rm MS}}(2\ {\rm GeV}) = 130 \pm 11 \pm 18\ {\rm MeV}, \tag{1}$$

where the first error is the statistical uncertainty and the second is the systematic uncertainty.

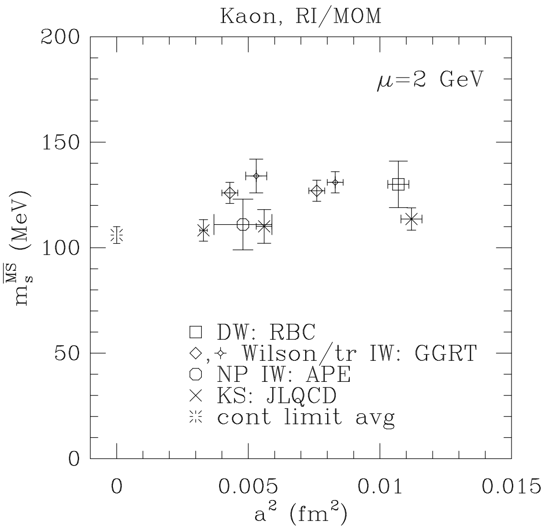

Figure 4. Strange quark mass vs. lattice spacing. Except for $a^2 = 0$ point, all m_s values computed using the ρ to set the scale, the K to set m_s, and the RI/MOM scheme. The $a^2 = 0$ asterisk is an average of three recent continuum limit calculations. See text for further description.

4. COMPARISON OF RESULTS

Fig. 4 shows several quenched lattice calculations of m_s plotted vs. lattice spacing squared. Except for the point at $a^2 = 0$, the quark masses were determined using the procedure described above and are plotted with statistical error bars only. The square corresponds to the DW fermion simulation of Ref. [2]. The crosses are from KS simulations [14], the big (small) diamonds are from Wilson (tree–level improved Wilson) simulations [15], and the octagon from a nonperturbatively improved Wilson action [16]. Finally, the asterisk displays the author's average of three recent continuum extrapolations of m_s, as described in the following paragraph.

Recently there have been three quenched lattice calculations of m_s which use nonperturbative renormalization and take the continuum limit: one with KS fermions [14] and two with nonperturbatively improved Wilson (NP IW) fermions [17,18], (see table below). In order to precisely compare among different lattice simulations which contain similar, if not identical, systematic effects, it is common practice to quote only statistical errors. However, before using the results phenomenologically, one must do one's best to include these systematic errors. As these simulations were performed on large volumes, the source of the largest systematic error (within the quenched approximation) is the ambiguity of which physical quantity is used to set the lattice spacing. Ref. [14] uses the lattice spacing set by the ρ mass, $a(M_\rho)$, while Refs. [17,18] use $a(f_K)$ (obtaining identical results with the lattice spacing set by the hadronic radius parameter r_0) and report that using $a(M_N)$ instead would increase m_s by 10 MeV. It is commonly seen in quenched lattice QCD that

$$a(f_K) < a(M_\rho) < a(M_N). \tag{2}$$

Since the latter two references quote a one–sided systematic uncertainty, the central value is shifted up by half of that uncertainty and the new error contains the statistical and systematic uncertainties added in quadrature. A 5% scale uncertainty is added to the statistical error of Ref. [14]. The weighted average of the three results is 106 ± 4 MeV.

Ref.	quoted m_s	a^{-1} syst.	adjusted m_s
[14]	106(7) MeV	± 5 MeV	106(9) MeV
[17]	97(4) MeV	+10 MeV	102(6) MeV
[18]	105(4) MeV	+10 MeV	110(6) MeV
	weighted average		106(4) MeV

Of course in nature there are sea quarks, so the quenched approximation could prove unreliable. Last year the CP-PACS collaboration reported deviations from experiment of the quenched hadron spectrum in the continuum limit at the level of 10% [19], one might expect the same level of disagreement for the strange quark mass. However, this year the CP-PACS collaboration has a preliminary result from two–flavor dynamical Wilson fermion simulations of $m_s^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 84 \pm 7$ MeV [20]. Although the mass renormalization has been computed only in (improved) perturbation theory, such a small strange quark mass is in conflict with lower bounds based on the positivity of hadronic spectral functions [21].

5. CONCLUSIONS

The RBC collaboration [1] has completed the calculation of the strange quark mass at a single gauge coupling $6/g_0^2 = 6.0$ using the domain wall fermion action. The mass renormalization is determined nonperturbatively through the RI/MOM scheme. The result, $130 \pm 11 \pm 18$ MeV, is consistent others at the same gauge coupling. Note this is a result at only one lattice spacing. Only in the continuum limit can a reliable comparison can be made to the NPSW and KS average of 106 ± 4 MeV. A final answer for m_s awaits several unquenched simulations exploring all the different systematic uncertainties already probed within the quenched approximation.

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